Revision 2 (Solutions)

Semester Two Examination

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

Student Number: In figures

In words

Teacher name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: Fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section Two: Calculator-assumed **65% (41 Marks)**

This section has **five (5)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 40 minutes.

Question 1 **(6 marks)**

The quadratic function $f(x) = ax^2 + bx + c$ passes through $P(5, 9)$ and has roots at $x = -4$ and $x = 7$.

(a) Determine the values of the constants a, b and c. (3 marks)

Solution
$f(x) = a(x+4)(x-7)$ $f(5) = 9 = a(9)(-2)$ $a = -\frac{1}{2}$ $f(x) = -\frac{1}{2}(x+4)(x-7)$ $= -\frac{1}{2}x^2 + \frac{3x}{2} + 14$ $a = -\frac{1}{2} = -0.5, \quad b = \frac{3}{2} = 1.5, \quad c = 14$
Specific behaviours
✓ writes in factored form ✓ uses given point to determine a ✓ expands and clearly states all values

(b) State the location of the y-intercept of the graph $y = -3f(x)$. (1 mark)

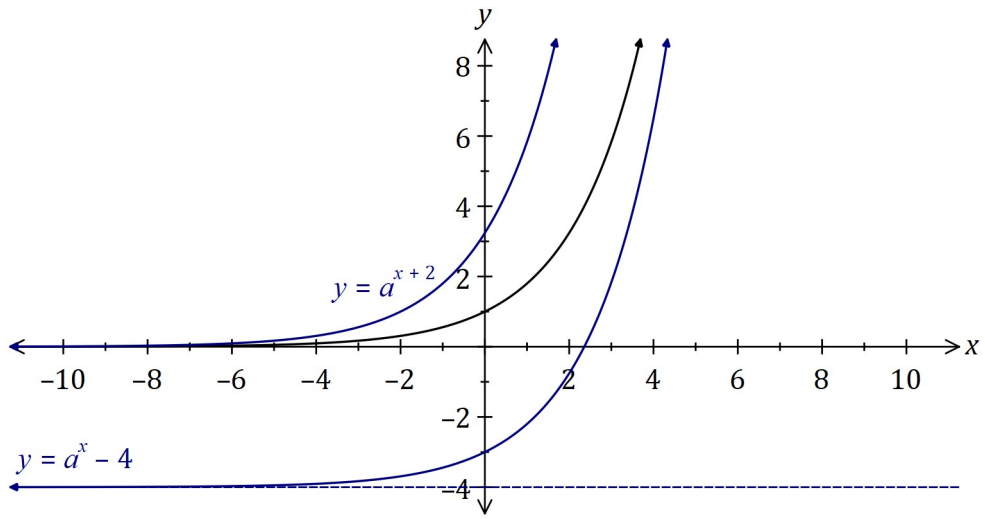
Solution
$y = -3(14) = -42$
Specific behaviours
✓ correct y-value

(c) State the location of the roots of the graph $y = f(4x)$. (2 marks)

Solution
$x = \frac{1}{4}(-4) = -1$ $x = \frac{1}{4}(7) = \frac{7}{4} = 1.75$
Specific behaviours
✓ uses correct horizontal dilation ✓ correct x-values

Question 2 **(8 marks)**

The graph of $y = a^x$ is shown below, where a is a positive constant.



(a) On the same axes, sketch and label the graphs of

(i) $y = a^{x+2}$. (2 marks)

(ii) $y = a^x - 4$. (2 marks)

Solution
See graph
Specific behaviours
(i) ✓ y-int close to (0, 3); ✓ touches x-axis close to (-7, 0) (ii) ✓ y-int at (0, -3); ✓ clear asymptote at $y = -4$

(b) The graph of $y = a^{x+3}$ intersects the graph of $y = 0.7^x$ when $x = -1.9$.

Determine, giving your answers to 3 significant figures,

(i) the y-coordinate of the point of intersection. (1 mark)

Solution
$y = 0.7^{-1.9}$ $= 1.9693\ldots$ $\approx 1.97\ \ (3\text{sf})$
Specific behaviours
✓ value that rounds to 1.97

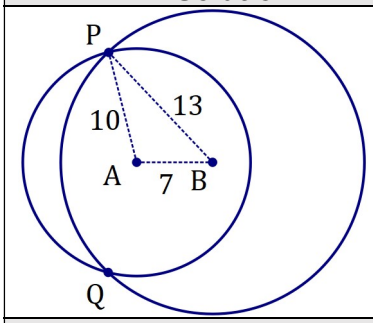
(ii) the value of the positive constant a. (3 marks)

Solution
$a^{-1.9+3} = 0.7^{-1.9}$ $a = 1.8516\ldots$ $a \approx 1.85\ \ (3\text{sf})$
Specific behaviours
✓ writes equation ✓ writes solution to equation ✓ rounds answers to (b)(i) & (ii) correctly

Question 3 **(11 marks)**

Two circles of radii 10 cm and 13 cm have centres at A and B respectively. The centres are 7 cm apart and the circles intersect at P and Q.

(a) Sketch a diagram of the two circles and clearly show triangle ABP. (2 marks)

Solution

Specific behaviours
✓ sketch showing A, B, P & Q ✓ shows triangle ABP

(b) Show that $\angle PBA = 49.6°$, when rounded to one decimal place. (2 marks)

Solution
$\cos B = \dfrac{7^2 + 13^2 - 10^2}{2(7)(13)}$ $\angle PBA = 49.583$ $= 49.6°\ (1\text{dp})$
Specific behaviours
✓ substitutes correctly into cosine rule ✓ states angle to 2 or more dp then rounds

(c) Determine the length of the chord PQ to the nearest millimetre. (2 marks)

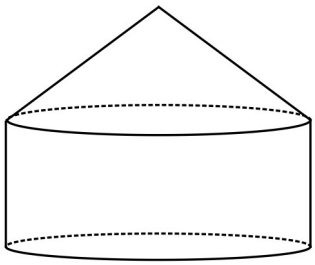
Solution
$\dfrac{1}{2}PQ = 13 \sin 49.6°$ $PQ = 19.8 \text{ cm}$
Specific behaviours
✓ uses right-angled trig ✓ states length, rounded to nearest mm

(d) Determine the area common to both circles. (5 marks)

Solution
Segment, centre B: $$\text{Angle: } 2 \times 49.58 \times \frac{\pi}{180} = 1.731$$ $$A = \frac{1}{2}(13)^2(1.731 - \sin(1.731)) = 62.83$$ Segment, centre A: $$\angle PAB = 98.21°$$ $$\text{Angle: } 2 \times 98.21 \times \frac{\pi}{180} = 3.428$$ $$A = \frac{1}{2}(10)^2(3.428 - \sin(3.428)) = 185.55$$ Total: $$A = 62.83 + 185.55 = 248.4 \text{ cm}^2$$
Specific behaviours
✓ uses segment formula with angles in radians ✓ states area of segment, centre B ✓ shows $\angle PAB$ ✓ states area of segment, centre A ✓ correct total area

Question 4 **(8 marks)**

A composite solid is made from a cone and a cylinder, both of height h cm and radius r cm, as shown below.



The dimensions are such that the sum of h and $3r$ is 36 cm.

(a) Show that the volume of the solid is given by $V = 48\pi r^2 - 4\pi r^3$. (3 marks)

Solution
$h + 3r = 36 \Rightarrow h = 36 - 3r$ $V = \frac{1}{3}\pi r^2 h + \pi r^2 h$ $= \frac{4}{3}\pi r^2 h$ $= \frac{4}{3}\pi r^2(36 - 3r)$ $= 48\pi r^2 - 4\pi r^3$
Specific behaviours
✓ writes h in terms of r ✓ substitutes into sum of cone and cylinder volumes ✓ simplifies

(b) Use differentiation to determine the values of r and h that will maximise the volume of the solid, and state this maximum volume. (5 marks)

Solution
$\frac{dV}{dr} = 96\pi r - 12\pi r^2$ $= 0$ when $r = 0, 8$ Optimum value of $r = 8$ cm $h = 36 - 3(8) = 12$ cm $V(8) = 1024\pi$ cm^3 (≈ 3217)
Specific behaviours
✓ differentiates ✓ determines root of derivative ✓ states optimum value of r ✓ calculates height h ✓ calculates volume V

Question 5 **(8 marks)**

Five different letters are selected from the eleven in the word COMRADESHIP. The order in which the letters are selected is not important, so that the selection COMRA is the same as the selection RAMOC, and so on.

(a) Determine the number of different selections

(i) of five letters. (2 marks)

Solution
$n = \binom{11}{5} = 462$
Specific behaviours
✓ uses combination ✓ correct number

(ii) of five letters that contain one vowel and four consonants. (2 marks)

Solution
$n = \binom{4}{1} \times \binom{7}{4} = 4 \times 35 = 140$
Specific behaviours
✓ splits selections ✓ multiplies selections

(b) Determine the probability that a random selection of five different letters

(i) includes the letters M and R. (2 marks)

Solution
$P = \dfrac{\binom{2}{2} \times \binom{9}{3}}{462} = \dfrac{84}{462} = \dfrac{2}{11}$
Specific behaviours
✓ selects (i) M & A (ii) other four ✓ states probability

(ii) includes at least one vowel. (2 marks)

Solution
$P = 1 - \dfrac{\binom{4}{0} \times \binom{7}{5}}{462} = 1 - \dfrac{21}{462} = \dfrac{441}{462} = \dfrac{21}{22}$
Specific behaviours
✓ selects no vowels ✓ states probability